Problem Set 5

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## Introduction

As an application of the some of the properties of expected values, this problem set steps through a proof that the expected value of the random variable that defines sample variance is the population variance, given that the population variance is defined.

For each of these questions, let be independent, identically distributed random variables with mean and variance .

Please complete the following tasks regarding the data in R. Please generate a solution document in R markdown and upload the .Rmd document and a rendered .doc, .docx, or .pdf document. Please turn in your work on Canvas.

These questions were rendered in R markdown through RStudio (<https://www.rstudio.com/wp-content/uploads/2015/02/rmarkdown-cheatsheet.pdf>, <http://rmarkdown.rstudio.com> ).

## Question 1 (10 points)

Let be independent, identically distributed random variables with mean and variance , and define the random variable by . Justify the equality

## Question 2 (10 points)

Let be independent, identically distributed random variables with mean and variance .

In terms of and , what is the value of ? Note that , while and . Please justify your answer.

Confirm numerically that your answer is correct for which has mean equal to and variance equal to .

f2<-function(x){x^2\*dexp(x,1/3)}  
integrate(f2,0,Inf)

## 18 with absolute error < 0.00054

## Question 3 (5 points)

Assuming that for all , what is . Recall that for any random variables with defined means.

## Question 4 (5 points)

Define the random variable by . What is the value of ? Please justify your answer.

Recall that the mean of equals and the variance equals . The fact that mentioned above may also be useful. Further, is constant with respect to the index in the sum.

## Question 5 (10 points)

Why is

## Question 6 (5 points)

Assuming that , that , and that , please simplify .

## Question 7 (5 points)

If , what is the value of ?